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SOUND TRANSMISSION THROUGH A THIN BAFFLED PLATE: VALIDATION OF A LIGHT FLUID APPROXIMATION WITH NUMERICAL AND EXPERIMENTAL RESULTS

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The response of a thin elastic rectangular baffled plate embedded in a gas and excited by an incident acoustic field is studied theoretically and experimentally. The model equations are analyzed by a perturbation method based on the expansion of the solution in eigenmodes and a small parameter. The small parameter expresses that the gas has a density which is small with respect to the density of the plate. The approximation obtained, the so-called *light fluid approximation*, is compared with the numerical solution of the exact boundary integral equations equivalent to the initial system of partial differential equations. Finally, the predictions given by the light fluid approximation are compared with experimental results. These two comparisons show the efficiency and the accuracy of the method.

1. INTRODUCTION

Problems of sound radiation or transmission by vibrating structures is one of the oldest problems that acousticians have tried to solve. Several very useful books, providing either analytical solutions of canonic problems or developing analytical and numerical methods have been published during the past 30 years. The oldest one to mention is due to Junger and Feit [1], which was published in 1972 by the Massachusetts Institute of Technology. Then a couple of books appeared during the 1980s [2,3] which proposed a more modern approach of vibro-acoustics: in particular, numerical methods (finite elements, boundary element methods, the statistical energy analysis method, etc) are widely developed. The most recent textbook has been published at the end of 1998 [4]: in our opinion, one of its main originalities is the use of functional analysis which is always required for a rigorous mathematical statement of mechanics problems.

A very fundamental review of the various problems raised by the coupling between a vibrating structure and a surrounding fluid is due to Crighton [5]. For a thin plate, the author made use of the following two intrinsic units: the inverse of the coincidence frequency (time unit) and the sound wavelength in the fluid at this frequency (length unit). The corresponding equations were called *non-dimensional equations*: we would prefer *reduced equations*, instead, because the number of independent variables on which the physical phenomenon depends is reduced to its minimum. The author mentioned that the reduced coupling parameter ε'—the ratio of the fluid density to the plate surface mass, both expressed in this units system—is dimensionless and less than unity for any data of practical interest: he suggested that this could be advantageous for developing numerical approximations. We do not totally agree with this conclusion.

In the present paper, we look for a numerical approximation of the solution as a Taylor-like series in terms of a small parameter ε truncated at the first order term, which is the ratio of the fluid density to the plate surface mass, both expressed in the same coherent system of units. There is a classical controversy on this point: it is often claimed that this parameter ε has a dimension—the inverse of a length—while ε' is dimensionless. In fact, the parameter that we use is dimensionless too. Indeed, the variables which appear in the mathematical equations which model a physical phenomenon have two different interpretations. The first one is that they represent the physical quantities and, thus, they have a dimension. If quantitative results are looked at, a system of units must be adopted: the choice of a coherent system of units—in the present case, a system based on a length unit, a mass unit and a time unit—leads to the same equations for the measures of the physical quantities in that units system. Thus, the corresponding equations are written and solved for dimensionless variables. As a conclusion, we can say that the only difference between the reduced equations and the general form used here is the choice of the system of units.

A second remark is that changing the units system has no advantage. If the Taylor-like series in terms of ε' truncated at the first order term provides an approximation with a relative error of α%, the Taylor-like series in terms of ε truncated at the first order term has exactly the same relative accuracy, whatever the system of units could be: this is due to the fact that the change of units system is a linear operation.

Finally, we will mention a paper by Norris *et al.* [6] in which a method much similar to ours is adopted.

In many situations of environmental noise pollution, the physical phenomenon involved is the radiation of an acoustic field by a vibrating structure embedded in air: that is, in a fluid with a density small compared to the structure one. This is the case in buildings where noise is transmitted through a solid structure (brick wall, glass, wood doors, etc.) which separates two rooms. Another example is the flow noise, inside a high-speed vehicle (car, train, plane) generated by the vibrations of its hull which is excited by a flow-induced external fluctuating wall pressure.

It is well known that the dynamic behaviour of a structure embedded in a gas is very close to the corresponding *in vacuo* one. Thus, in most situations, the presence of the surrounding gas is a small perturbating factor. The main influence of this factor is an increase of the damping of the *in vacuo* resonance modes of the structure. For the prediction of this vibro-acoustic behaviour, it is possible to take advantage of the existence of a small factor by using a perturbation method.

In a recent paper [7], the general problem of a fluid-loaded plate has been studied in some detail. Mainly three types of representation of solutions were considered: boundary integral expressions, fluid-loaded eigenmode series representation and resonance mode series expansion. For each of the corresponding equations, a perturbation method has been developed. A rigorous presentation of the eigenmode and resonance mode approaches, mathematically justified and with a physical interpretation was developed in reference [8] for the case of a three-dimensional elastic body. It is assumed here that these results remain true for the thin-plate approximation of the elasticity equations.

In the present paper, we consider the eigenmode series representation of the solution. In section 2, the problem is stated, the eigenmode series representation is briefly recalled and the perturbation method which gives the fluid-loaded eigenmodes and eigenfrequencies is summarized. The section 3 is devoted to the comparison between the light fluid approximation and the numerical solution of the exact boundary integral equations. The section 4 is devoted to the comparison of the perturbation approximation with experimental results: the experiments have been conducted on a stainless-steel plate mounted in a wall which separates two large-size anechoïc rooms.

2. STATEMENT OF THE PROBLEM

Consider a thin elastic plate, with thickness h, occupying the domain Σ of the plane $z = 0$ in a three-dimensional space. Along its boundary $\partial\Sigma$, an external unit normal vector $\mathbf{n}$ can be defined almost everywhere. The baffle, which occupies the complement Σ' of $\bar{\Sigma} = \Sigma \cup \partial\Sigma$, is perfectly rigid. The plate thickness is negligible compared to the wavelengths involved. The two half-spaces $\Omega^+(z > 0)$ and $\Omega^-(z < 0)$ contain a perfect gas.

The mechanical characteristics of the plate are: E = Young's modulus, ν = the Poisson ratio, $D = Eh^3/12(1 - \nu^2)$ = rigidity, μ = plate mass per unit area. The plate is assumed to be clamped along $\partial\Sigma$. The damping of the plate material can easily be introduced by an imaginary part in the Young's modulus: this is a correct modelization of elastic waves absorption in many standard materials, in particular in metals for which the relative value of the Young's modulus imaginary part is $\mathcal{O}(10^{-4})$. The fluid is characterized by a density μ_0 and a sound speed c_0. The system is excited by a harmonic acoustic source, $S^-(Q)e^{-i\omega t}$, located in Ω^-.

2.1. GOVERNING EQUATIONS

Let $u(M) = u(x, y)$ be the plate displacement, positive in the $z > 0$ direction; and let $p^+(Q) = p^+(x, y, z)$ and $p^-(Q) = p^-(x, y, z)$ denote the acoustic pressure fields, respectively, in Ω^+ and Ω^-. The pressure step $P(Q) = P(x, y)$ across the plate is defined by

$$P(x, y) = \lim_{\imath \to \infty} [p^+(x, y, \imath) - p^-(x, y, -\imath), \quad \imath > 0.$$

The functions $u(M)$, $p^+(Q)$ and $p^-(Q)$ satisfy the following system of equations:

$$(\Delta + k^2)p^+(Q) = 0, \quad Q \in \Omega^+,$$

$$(\Delta + k^2)p^-(Q) = S^-(Q), \quad Q \in \Omega^-,$$

$$(D\Delta^2 - \mu\omega^2)u(M) + P(M) = 0, \quad M \in \Sigma,$$

$$\partial_z p^+(M) = \partial_z p^-(M) = \begin{cases} \omega^2 \mu_0 u(M) & M \in \Sigma, \\ 0, & M \in \Sigma', \end{cases}$$

$$u(M) = \partial_n u = 0, \quad M \in \partial\Sigma \tag{1}$$

and a Sommerfeld condition on p^+ and p^-.

2.2. GREEN'S REPRESENTATION OF THE PRESSURE FIELDS AND THE INTEGRO-DIFFERENTIAL EQUATION FOR THE PLATE DISPLACEMENT

Let $\mathscr{G}_\omega(Q, Q')$ be the Green function of the Helmholtz equation which satisfies a homogeneous Neumann condition on $z = 0$ and the Sommerfeld condition at infinity; that is

$$\mathscr{G}_\omega(Q, Q') = -\frac{e^{ikr(Q,Q')}}{4\pi r(Q, Q')} - \frac{e^{ikr(Q,Q'_-)}}{4\pi r(Q, Q'_-)},$$

where the co-ordinates of the points Q' and Q'_- are, respectively, (x', y', z') and $(x', y', -z')$.

The pressure fields can be written as

$$p^+(Q) = \omega^2 \mu_0 \int_\Sigma u(M')\mathscr{G}_\omega(Q, M')\,d\sigma(M'), \quad Q \in \Omega^+,$$

$$p^-(Q) = p_0^-(Q) - \omega^2 \mu_0 \int_\Sigma u(M')\mathscr{G}_\omega(Q, M')\,d\sigma(M'), \quad Q \in \Omega^-,$$

with

$$p_0^-(Q) = \int_{\Omega^-} \mathscr{G}_\omega(Q, Q')S^-(Q')\,dQ'. \tag{2}$$

By introducing equation (2) into the third of equations (1), one gets the well-known integro-differential equation for the plate displacement:

$$(D\Delta^2 - \mu\omega^2)u(M) + 2\mu_0\omega^2 \int_\Sigma u(M')\mathscr{G}_\omega(M, M')\,\mathrm{d}\sigma(M') = p_0^-(Q), \quad M \in \Sigma \quad (3)$$

with the boundary conditions of clamping. As done in reference [7], this equation can be replaced by its weak form (energy form), i.e.,

$$a(u, v) - \mu\omega^2\{(u, v) - \varepsilon\beta_\omega(u, v)\} = \langle p_0^-, v\rangle \quad (4)$$

with

$$\varepsilon = 2(\mu_0/\mu)$$

$$\langle u, v\rangle = \int_\Sigma u(M)v^*(M)\,\mathrm{d}\sigma(M),$$

$$a(u, v) = D\int_\Sigma \left\{\Delta u \Delta v^* + (1 - v)\left[2\frac{\partial^2 u}{\partial x\partial y}\frac{\partial^2 v^*}{\partial x\partial y} - \frac{\partial^2 u}{\partial x^2}\frac{\partial^2 v^*}{\partial y^2} - \frac{\partial^2 u}{\partial y^2}\frac{\partial^2 v^*}{\partial x^2}\right]\right\}\mathrm{d}\sigma(M)$$

$$\beta_\omega(u, v) = \int_\Sigma\int_\Sigma u(M)\mathscr{G}_\omega(M, M')v^*(M')\,\mathrm{d}\sigma(M)\,\mathrm{d}\sigma(M').$$

Here $v(M)$ is an appropriate test function, and $v^*(M)$ is its complex conjugate. One can recall that if v is replaced by u, the first integral is proportional to the kinetic energy of the plate, the second one is its potential energy, and the third one is proportional to the radiated energy.

The definition of the appropriate test function space is classical (see any elementary textbook on functional analysis or reference [4]). Simply recall that the functions v must have the same regularity as the plate displacement u and satisfy the same boundary conditions: the most classical set of test functions is generated by the *in vacuo* resonance modes of the plate.

3. EIGENMODE SERIES REPRESENTATION OF THE SOLUTION AND LIGHT FLUID APPROXIMATION

3.1. REPRESENTATION OF THE SOLUTION AS A FLUID-LOADED PLATE EIGENMODE SERIES

The eigenmodes U_n and eigenvalues Λ_n of the fluid-loaded plate are defined by the eigenvalue problem

$$a(U_n, v) = \Lambda_n\{\langle U_n, v\rangle - \varepsilon\beta_\omega(U_n, v)\}, \quad (5)$$

where U_n is sought in a suitable functional space: in particular, every function U of this space satisfies the boundary conditions of clamping

$$U(M) = \partial_n U(M) = 0, \quad M \in \Sigma.$$

It can be shown that the U_n satisfy the following orthogonality relationships:

$$\langle U_n, U_m^* \rangle - \varepsilon\beta_\omega(U_n, U_m^*) = 0 \quad \text{for } m \neq n,$$

or, equivalently

$$a(U_n, U_m^*) = 0 \quad \text{for } m \neq n.$$

A pressure field can be associated with each eigenmode:

$$p_n(Q) = \omega^2 \mu_0 \int_\Sigma U_n(M')\mathcal{G}_\omega(Q, M')\,\mathrm{d}\sigma(M'), \quad Q \in \Omega^\pm. \tag{6}$$

For simplicity, it is assumed that only simple eigenvalues are present. The response of the system can, thus, be expanded into the following series of U_n and p_n:

$$u(M) = \sum_{n=1}^{\infty} \frac{\Lambda_n}{\Lambda_n - \mu\omega^2} \frac{\langle p_0^-, U_n^* \rangle}{a(U_n, U_n^*)} U_n(M), \quad M \in \Sigma,$$

$$p^+(Q) = \sum_{n=1}^{\infty} \frac{\Lambda_n}{\Lambda_n - \mu\omega^2} \frac{\langle p_0^-, U_n^* \rangle}{a(U_n, U_n^*)} p_n(Q), \quad Q \in \Omega^+,$$

$$p^-(Q) = p_0^-(Q) - \sum_{n=1}^{\infty} \frac{\Lambda_n}{\Lambda_n - \mu\omega^2} \frac{\langle p_0^-, U_n^* \rangle}{a(U_n, U_n^*)} p_n(Q), \quad Q \in \Omega^-, \tag{7}$$

These series are defined for any real angular frequency ω, because the eigenvalues Λ_n have a non-zero imaginary part. When an eigenvalue has a multiplicity order equal to or greater than 2, the modification of the former formulas is performed in the classical way (see, for example, reference [9]).

3.2. LIGHT FLUID APPROXIMATION OF THE EIGENMODE SERIES

Because the fluid is a gas, the ratio $\varepsilon = 2\mu_0/\mu$ is, in general, a small parameter, and an appropriate perturbation method can be used. The eigenvalues and eigenmodes are expanded with respect to ε: it is hoped that the sum of the first two terms (zero and first orders) provide a good approximation. The authors have followed the method developed in reference [10] called *the Rayleigh–Schrödinger method* by Nayfeh. An approach to the perturbation methods, much similar to

Nayfeh's one, is presented in reference [11] where the formalism of functional analysis is emphasized.

Let us briefly recall the basic principle. The eigenvalues Λ_n and the eigenmodes U_n are expanded as follows:

$$U_n = U_n^0 + \varepsilon U_n^1 + \varepsilon^2 U_n^2 + \cdots \qquad \Lambda_n = \Lambda_n^0 + \varepsilon \Lambda_n^1 + \varepsilon^2 \Lambda_n^2 + \cdots .$$

This expansion is introduced into equation (5) and the terms with like power of ε are made equal. This leads to a countable set of equations, the zero and first order ones being

$$a(U_n^0, v) = \Lambda_n^0 \langle U_n^0, v \rangle,$$

$$a(U_n^1, v) = \Lambda_n^1 \langle U_n^0, v \rangle - \Lambda_n^0 \beta_\omega (U_n^0, v) + \Lambda_n^0 \langle U_n^1, v \rangle. \tag{8}$$

The solution of the zeroth order equation is the nth *in vacuo* eigenvalue and eigenmode. By setting $v = U_n^{0*}$ in the second equation (8), it easily follows that the first order equation leads to

$$\Lambda_n^1 = \Lambda_n^0 \frac{\beta_\omega(U_n^0, U_n^{0*})}{\langle U_n^0, U_n^{0*} \rangle} = \Lambda_n^0 \frac{\int_\Sigma \int_\Sigma U_n^0(M) \mathscr{G}_\omega(M, M') U_n^0(M') \, \mathrm{d}\sigma(M) \, \mathrm{d}\sigma(M')}{\int_\Sigma U_n^0(M)^2 \, \mathrm{d}\sigma(M)}, \tag{9}$$

$$U_n^1(M) = - \sum_{q=1,\, q \neq n}^{\infty} \frac{\Lambda_n^0}{\Lambda_q^0 - \Lambda_n^0} \frac{\beta_\omega(U_n^0, U_q^{0*})}{\langle U_q^0, U_q^{0*} \rangle} U_q^0(M)$$

$$= - \sum_{q=1,\, q \neq n}^{\infty} \frac{\Lambda_n^0}{\Lambda_q^0 - \Lambda_n^0} \frac{\int_\Sigma \int_\Sigma U_n^0(M) \mathscr{G}_\omega(M, M') U_q^0(M') \, \mathrm{d}\sigma(M) \, \mathrm{d}\sigma(M')}{\int_\Sigma U_q^0(M)^2 \, \mathrm{d}\sigma(M)} U_q^0(M). \tag{9a}$$

The corresponding light fluid approximation of the system response is given by

$$u(M) \simeq \sum_{n=1}^{\infty} \frac{\Lambda_n^0 + \varepsilon \Lambda_n^1}{\Lambda_n^0 + \varepsilon \Lambda_n^1 - \mu\omega^2} \frac{\langle p_0^-, U_n^{0*} + \varepsilon U_n^{1*} \rangle}{a(U_n^0 + \varepsilon U_n^1, U_n^{0*} + \varepsilon U_n^{1*})} [U_n^0(M) + \varepsilon U_n^1(M)], \tag{10}$$

$$p^+(Q) \simeq \sum_{n=1}^{\infty} \frac{\Lambda_n^0 + \varepsilon \Lambda_n^1}{\Lambda_n^0 + \varepsilon \Lambda_n^1 - \mu\omega^2} \frac{\langle p_0^-, U_n^{0*} + \varepsilon U_n^{1*} \rangle}{a(U_n^0 + \varepsilon U_n^1, U_n^{0*} + \varepsilon U_n^{1*})} \hat{p}_n(M),$$

$$p^-(Q) \simeq p_0^-(Q) - \sum_{n=1}^{\infty} \frac{\Lambda_n^0 + \varepsilon \Lambda_n^1}{\Lambda_n^0 + \varepsilon \Lambda_n^1 - \mu\omega^2} \frac{\langle p_0^-, U_n^{0*} + \varepsilon U_n^{1*} \rangle}{a(U_n^0 + \varepsilon U_n^1, U_n^{0*} + \varepsilon U_n^{1*})} \hat{p}_n(M), \tag{10a}$$

with

$$\hat{p}_n(M) = \omega^2 \mu_0 \int_\Sigma [U_n^0(M') + \varepsilon U_n^1(M')] \mathscr{G}_\omega(M, M') \, \mathrm{d}\sigma(M').$$

This result can be compared with the type of approximation—outer and inner expansions—which is developed in reference [6]. Away from any *in vacuo* resonance frequency of the plate, ε can be made equal to 0 in expressions (10) and (10a), and one obtains the first two terms of the outer expansion. If ε is made equal to 0, these expressions are not defined at each *in vacuo* resonance frequency of the plate, as is the outer expansion of the cited article. In the neighborhood of *in vacuo* resonance frequency of the plate, these expressions correspond to the inner expansion.

Let us assume now that there exists one *in vacuo* eigenvalue with multiplicity order 2: for example $\Lambda_r = \Lambda_{r+1}$. The corresponding eigenmodes U_r and U_{r+1} are independent functions which can be assumed to be orthogonal. Upon using the perturbation expansions of these eigenmodes and two different perturbation expansions of the double eigenvalue, it appears that equations (8) stand for $\{U_r, \Lambda_r\}$ and $\{U_{r+1}, \Lambda_{r+1}\}$. As a consequence, equalities (9) and (9a) are still valid, but the correcting terms Λ_r^1 and Λ_{r+1}^1 are *a priori* different. Expressions (10) and (10a) remain unchanged.

It must be mentioned that this approximation is valid under restrictive conditions: the first one is that the parameter ε is small compared to unity ($\varepsilon < 0{\cdot}4\ \mathrm{m}^{-1}$ seems quite correct; it is less obvious to give a criterium in reduced units). A second condition of validity is that the driving frequency must be away from the critical frequency (frequency for which the wavenumber in the plate is equal to the wavenumber in the fluid). In reference [7] it was suggested that the validity conditions of the light fluid approximation do not depend on the plate geometry and/or on the boundary conditions, and are the same for a bounded or an unbounded plate. If this assumption is adopted, the validity conditions to account for are those which apply for the simple situation of an infinite plate.

4. EFFICIENCY OF THE LIGHT FLUID APPROXIMATION

A computer program has been developed for the case of a rectangular clamped baffled plate [12]. The *in vacuo* eigenfrequencies and eigenmodes used in the light fluid approximation are calculated by Warburton's method [13].

The accuracy and efficiency of the light fluid approximation is validated in two different ways: in both cases, the system is excited by an incident spherical wave. In the first case, one compares the approximate sound level attenuation as obtained by the light fluid approximation with the numerical solution of the exact equations. In the second case, the approximate sound level attenuation is compared with experimental results.

4.1. COMPARISON BETWEEN THE LIGHT FLUID APPROXIMATION AND THE NUMERICAL SOLUTION OF THE EXACT EQUATIONS

The exact equations are solved by the method proposed in reference [14]. The system of partial differential equations is transformed into a system of boundary integral equations for the plate displacement u, the pressure step P and two-layer

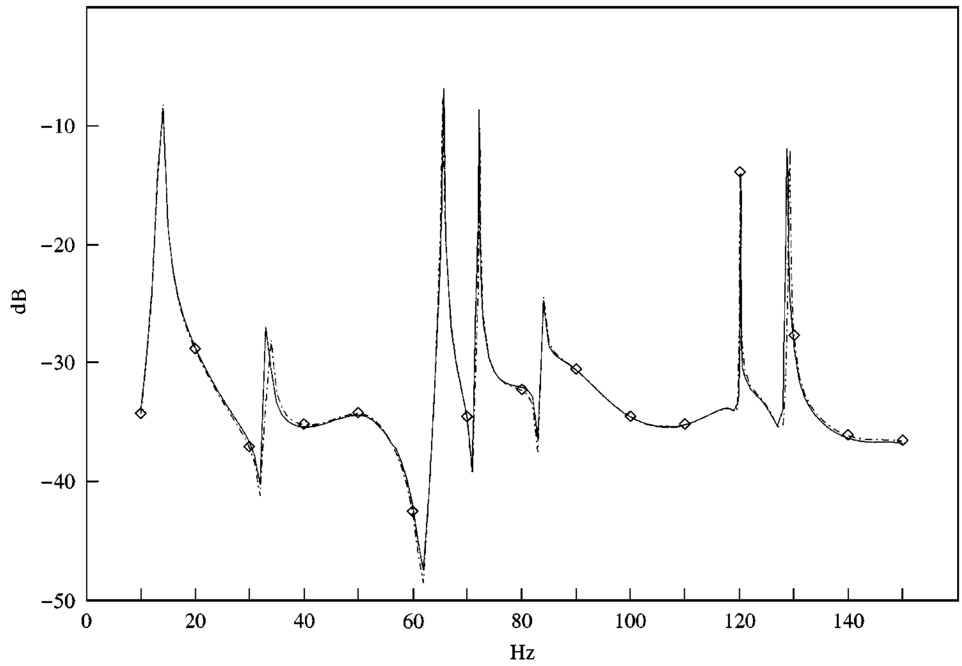

Figure 1. Comparison between the light fluid approximate transfer function (–··–◇–··–) and the "exact" one (——): steel plate with thickness 2 mm, occupying $\Sigma = (-0{\cdot}77\ \text{m} < x < 0{\cdot}77\ \text{m}, -0{\cdot}50\ \text{m} < y < 0{\cdot}50\ \text{m})$ in the plane $z = 0$; source at $x = 0$, $y = -1{\cdot}3$ m, $z = -7{\cdot}3$ m; first "microphone" at $x = 0$, $y = 0$, $z = -0{\cdot}5$ m; second "microphone" at $x = 0$, $y = 0$, $z = 0{\cdot}25$ m.

densities along $\partial\Sigma$ to account for the boundary conditions. The unknown functions are approximated by truncated series of Techebycheff polynomials. The coefficients are solutions of a collocation system of linear algebraic equations which is equivalent to a Ritz–Galerkin system.

The plate material has the following characteristics: $E = 2{\cdot}1 \times 10^{11}$ Pa, $\nu = 0{\cdot}33$, $\mu = 15{\cdot}6\ \text{kg/m}^2$. The fluid physical properties are $c_0 = 340$ m/s, $\mu_0 = 1{\cdot}29\ \text{kg/m}^3$. The wavelength at the coincidence frequency is $\lambda_c = 0{\cdot}0586$ m.

The plate dimensions are: $(-0{\cdot}77\ \text{m} < x < 0{\cdot}77\ \text{m}, -0{\cdot}55\ \text{m} < y < 0{\cdot}55\ \text{m})$; in reduced units, one has: $(-13{\cdot}14\lambda_c < \bar{x} < 13{\cdot}14\lambda_c, -9{\cdot}386\lambda_c < \bar{y} < 9{\cdot}386\lambda_c)$.

The coupling parameter is equal to $0{\cdot}165\ \text{m}^{-1}$, or, in reduced units, to $0{\cdot}0970\ \lambda_c^{-1}$. The point source is located at ($x = 0$, $y = -1{\cdot}3$ m, $z = -7{\cdot}3$ m). The difference between the sound levels at points ($x = 0$ m, $y = 0$ m, $z = 0{\cdot}25$ m) and at ($x = 0$ m, $y = 0$ m, $z = -0{\cdot}5$ m) is calculated by both programs. The results are presented in Figure 1.

The frequency step is 1 Hz in the regular parts of the curves and 0·1 Hz around the peaks. The agreement between the two curves is excellent: the relative difference between resonance frequencies is $\mathcal{O}(10^{-3})$ and the level difference is less than 1 dB (let us recall that the relative accuracy of Warburton's approximation of the resonance frequencies is $\mathcal{O}(10^{-3})$). Other calculations have been conducted for various examples and show a similar agreement.

4.2. COMPARISON BETWEEN THE LIGHT FLUID PREDICTIONS AND EXPERIMENT

The experiment has been conducted in the twin anechoic rooms of the Laboratoire de Mécanique et d'Acoustique. A large anechoic room is connected to a smaller semi-anechoic room by an aperture in which the plate is clamped. The wall between the two rooms is bare on the semi-anechoic side (almost perfectly rigid surface) and covered with glass-wool wedges on the other side (almost perfectly absorbent baffle). The sound source is located in the semi-anechoic room. Thus, the experimental conditions are somewhat different from the model problem of a perfectly rigid baffle on both sides. But it can easily be seen analytically that the transmitted sound field close to the central part of the plate is not very much influenced by the absorbing properties of the baffle.

Indeed, the transmitted pressure field has the following Green's representation,

$$p^+(Q) = \omega^2 \mu_0 \int_\Sigma u(M') \mathscr{G}_\omega(Q, M') \, \mathrm{d}\sigma(M') + \int_{\Sigma'} \partial_{z'} p(M') \mathscr{G}_\omega(Q, M') \, \mathrm{d}\sigma(M'),$$

where the function $\partial_{z'} p(M')$ is determined by a boundary integral equation expressing, for example, that a Robin condition is satisfied along Σ':

$$\partial_{z'} p(M) + \frac{\mathrm{i}k}{\zeta} \int_{\Sigma'} \partial_{z'} p(M') \mathscr{G}_\omega(Q, M') \, \mathrm{d}\sigma(M') = -\frac{\mathrm{i}k}{\zeta} \omega^2 \mu_0 \int_\Sigma u(M') \mathscr{G}_\omega(Q, M') \, \mathrm{d}\sigma(M').$$

The first term in the representation of $p^+(M)$ can be considered as an incident field, the source being the vibrating plate. The second term is a diffracted field, the incident field being the plate radiation: it corresponds to a single-layer potential with a source lying on the baffle. As a consequence, close to the plate (energy source), the first term must be predominant, at least because the secondary source corresponding to the second term lies in the exterior of the plate. Furthermore, the former boundary integral equation shows that this term is a secondary radiation: the function $\partial_{z'} p(M')$ can be calculated by a classical convergent iterative procedure which leads to $\partial_{z'} p(M') = 0$ as zero order approximation. This conclusion is, of course, independent of the fluid density.

The experimental plate is made of stainless steel with the dimensions used in the former subsection. Its thickness has been measured at more than 25 points and a mean value $h = 0{\cdot}0019$ m has been deduced (the thickness fluctuations are about $\pm 10^{-4}$ m).

The rigidity has been determined experimentally by the following procedure. The first 10 resonance frequencies of a circular plate with free boundary are measured and the rigidity is adjusted to get the best fit with the theoretical ones: this leads to $D = 122$ N × m. Practically, the resonance frequencies of the circular plate are measured in air. Nevertheless, they are considered as equal to the *in vacuo* ones, because the main influence of the air embedding is the damping of the resonances, the shifting of the real part being a few percent (much less than the error due to the

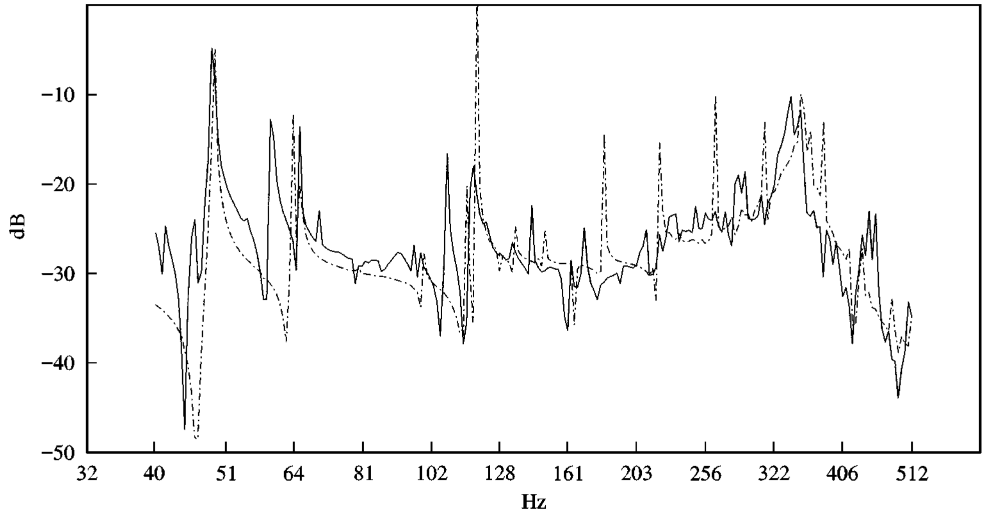

Figure 2. Comparison between the measured transfer function (——) and its *light fluid approximation* (–·–·–).

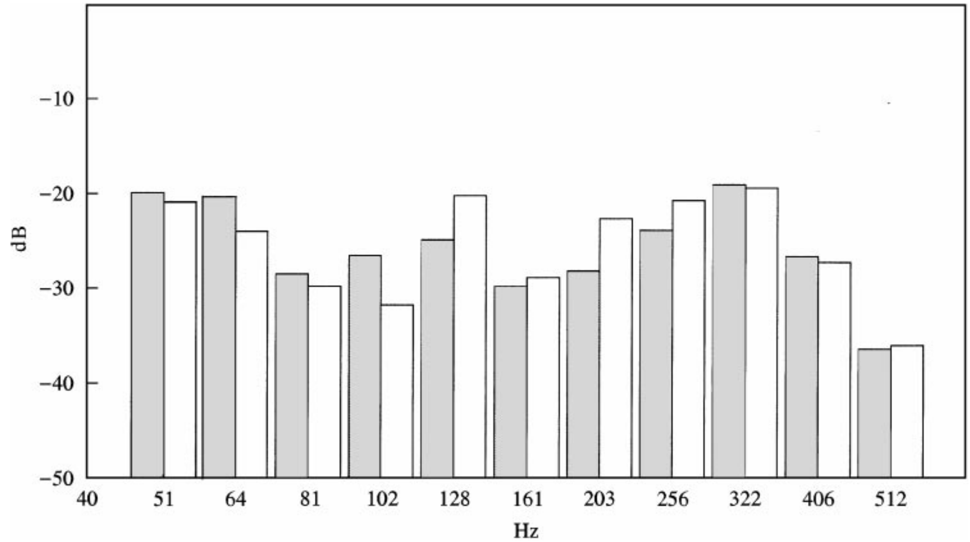

Figure 3. Comparison between the measured mean (third octaves) transfer function ▨ and its *light fluid approximation* □.

lack of the material isotropy or the thickness fluctuations). The Poisson ratio has been taken equal to 0·33.

The sound source is located at ($x = 0{\cdot}26$ m, $y = -0{\cdot}17$ m, $z = -3{\cdot}0$ m); a first microphone, which is located at ($x = -0{\cdot}26$ m, $y = -0{\cdot}17$ m, $z = -0{\cdot}25$ m), records the sum of the incident and reflected fields; a second microphone, which is located at ($x = -0{\cdot}26$ m, $y = -0{\cdot}17$ m, $z = 0{\cdot}25$ m), records the transmitted field.

The transfer function between the second and the first microphones are measured and computed as functions of the frequency. Figure 2 shows the result obtained with a frequency step of 1 Hz and Figure 3 presents a third-octave mean transfer function.

5. CONCLUSION AND FINAL REMARKS

The comparison between the light fluid approximation and the experimental results shows the efficiency of this theoretical approach. The first two sources of error in the numerical model are the non-isotropy and the thickness variations of the plate. A third source is that the plate is not an absolutely plane surface: it is, in fact, a very shallow shell (the central deflection is about 1 cm), with boundaries slightly constrained. These discrepancies between the theoretical model and the experimental device can explain the shifts between the resonance frequencies. The resonance peaks of the computed curve are, in general, sharper and higher than the experimental ones: this is due to the fact that the stainless steel has been considered as perfectly elastic. If a small damping is introduced (for example by using a complex Young's modulus $E(1 + i\varepsilon)$ with $10^{-4} < \varepsilon < 10^{-3}$) the peaks levels are much reduced. But, for this particular experiment, it does not seem very useful to work out a much more accurate theoretical modelling; indeed, from a practical point of view, the most significant acoustical properties of the plate are described by its third-octave mean response which is not very much affected by all these modelling errors.

In the present work, the light fluid approximation has been developed for the representation of the solution as a series of the fluid-loaded plate eigenmodes. As shown in reference [7], a light fluid approximation can be established for any representation of the solution of the vibro-acoustics problem. In practice, such an approximation is based on the analytical or numerical solutions of two disjoint problems: on the one hand, the response of the *in vacuo* structure; on the other hand, the non-homogeneous Neumann problem for the Helmholtz equation. At each step of the perturbation method, these two problems are solved alternatively.

In our opinion, the perturbation method is efficient if the first correcting order term is sufficient to provide the accuracy required for the real-life situation under consideration. Indeed, terms of higher orders imply to compute integrals over the structure surface which involve the iterated Green's kernel. Thus, to our knowledge, it is faster to solve the exact equations than to compute the approximate solution up to order 2. Nevertheless, at least for some simple geometries, it is, perhaps, possible to develop an integration algorithm which, being well adapted to the functions to integrate, will be particularly efficient. But this is a matter of numerical techniques that the authors have not entered in, up to now.

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